Closing tonight: 2.7-8
Closing Wed: 2.8
Closing Fri: 3.1-2
Visit office hours 1:30-3:00pm in PDL C-339

Today: Finish 2.8, start 3.1/3.2.

Entry Task: **Algebra Skills Test**

Rewrite each of the following in the form: $a\,x^b$

1. $7\sqrt{x^3}$
2. $\dfrac{13}{2x^6}$
3. $\dfrac{32 \cdot 15\,x^4}{16 \cdot 5\,x^6}$
4. $\dfrac{x^7\sqrt{x}}{4(x^2)^3}$
5. $17\sqrt[5]{x^3}$

2.8: Differentiability

Sometimes we can have a place where "slope of tangent" doesn't make sense.

Definition: We say a function, $y = f(x)$ is **differentiable** at $x = a$ if the following limit exists:

$$\lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

Otherwise it is not differentiable at a.

In order to be differentiable:

1. It must be defined at x = a.
2. It must be continuous at x = a.
3. The "slope" must be the same from both sides.

3.1/3.2 Intro to Derivative Rules

Some Basic Limit Laws:

1. $\frac{d}{dx}(c) = 0$

2. $\frac{d}{dx}(f(x) + g(x)) = f'(x) + g'(x)$

3. $\frac{d}{dx}\left(cf(x)\right) = cf'(x)$

4. $\frac{d}{dx}(x^n) = nx^{n-1}$

5. $\frac{d}{dx}(e^x) = e^x$

$\frac{d}{dx}(a^x) = a^x \ln(a)$

Definition of Derivative:

1. Constant Rule: For $f(x) = c$,

$$\lim_{h\to 0}\frac{f(x+h) - f(x)}{h} = \lim_{h\to 0}\frac{c-c}{h} = 0.$$

2. Sum rule:

$$\lim_{h\to 0}\frac{f(x+h) + g(x+h) - f(x) - g(x)}{h} =$$

$$\lim_{h\to 0}\frac{f(x+h) - f(x)}{h} + \lim_{h\to 0}\frac{g(x+h) - g(x)}{h}$$

3. Constant coefficient rule:

$$\lim_{h\to 0}\frac{cf(x+h) - cf(x)}{h} = c\lim_{h\to 0}\frac{f(x+h) - f(x)}{h}$$

4. Power Function Rule: For $f(x) = x^n$,

$$\lim_{h\to 0}\frac{f(x+h) - f(x)}{h} = \lim_{h\to 0}\frac{(x+h)^n - x^n}{h}$$

5. Exponential Function Rule:
For $f(x) = a^x$,

$$\lim_{h\to 0}\frac{f(x+h) - f(x)}{h} = \lim_{h\to 0}\frac{a^{x+h} - a^x}{h}$$

$$= a^x \lim_{h\to 0}\frac{a^h - 1}{h}$$

$$6. \frac{d}{dx}(f(x)g(x)) = f(x)g'(x) + f'(x)g(x)$$

$$7. \frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$$

6. Product Rule:

$$\begin{aligned}
&\lim_{h\to 0}\frac{f(x+h)g(x+h) - f(x)g(x)}{h} \\
&\quad = \lim_{h\to 0}\frac{f(x+h)g(x+h) - f(x+h)g(x) + f(x+h)g(x) - f(x)g(x)}{h} \\
&\quad = \lim_{h\to 0} f(x+h)\frac{g(x+h) - g(x)}{h} + g(x)\lim_{h\to 0}\frac{f(x+h) - f(x)}{h}
\end{aligned}$$